

Newton's Law of Cooling

Newton's Law of Cooling states that the rate of change of temperature of a body is proportional to the difference between the temperature of that body and that of the surrounding medium. Let the temperature of the body be $T(t)$ at time t . If T_0 denotes the initial temperature of the body at time 0 and T_a denotes the temperature of the surrounding medium, then $dT/dt = -k(T - T_a)$, where the negative sign is for the case where the body is getting cooler. The solution of the differential equation is $T(t) = T_a + (T_0 - T_a)e^{-kt}$.

Example: A dead body was 80 degrees when it was discovered. Two hours later it had cooled to 75 degrees. The room is 60 degrees. What time did the body die?



Time	Temp
0	80
a	60
2	75

Assume the body was 98.6 degrees at time of death. Find t so Temp = 98.6

$$T = T_a + (T_0 - T_a)e^{-kt}$$

Step One: solve for k :

$$75 = 60 + (80 - 60) * e^{-2k}$$

$$15 = 20 * e^{-2k}$$

$$15 / 20 = e^{-2k}$$

$$\text{LN}(15 / 20) = \text{LN}(e^{-2k}) = -2k$$

$$k = -0.2877 / -2$$

$$k = 0.1439$$

Step Two: solve for t :

$$T(t) = 60 + 20 * e^{-0.1439t}$$

$$98.6 = 60 + 20 * e^{-0.1439t}$$

$$38.6 = 20 * e^{-0.1439t}$$

$$38.6 / 20 = e^{-0.1439t}$$

$$\text{LN}(38.6 / 20) = \text{LN}(e^{-0.1439t}) = -0.1439t$$

$$t = 0.6575 / -0.1439$$

$$t = -4.5691 \approx 4.57 \text{ hours before the body was found}$$